J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

FYJC FINAL EXAM - 04

SOLUTION SET

DURATION - 2 1/2 HR   MARKS - 80

Q-1

01. find the range of the given function :
$f(x) = 9 - 2x^2 \; ; \quad -5 \le x \le 3$

$-5 \le x \le 3$

$0 \le x^2 \le 25$

$0 \le 2x^2 \le 50$

$0 \ge -2x^2 \ge -50$

$9 \ge 9-2x^2 \ge -50 + 9$

$9 \ge f(x) \ge -41$

Range of f(x) : [−41,9]

02. $\lim_{x\to 0} \dfrac{\cos x - \cos^2 x}{x^2}$

$= \lim_{x\to 0} \dfrac{\cos x\,(1 - \cos x)}{x^2}$

$= \lim_{x\to 0} \dfrac{\cos x \;\; 2\sin^2(x/2)}{x^2}$

$= \lim_{x\to 0} 2 \;\cos x \left(\dfrac{\sin(x/2)}{x}\right)^2$

$= \lim_{x\to 0} 2 \;\cos x \left(\dfrac{1}{2}\,\dfrac{\sin(x/2)}{(x/2)}\right)^2$

$= 2 \;\cos 0 \left(\dfrac{1(1)}{2}\right)^2$

$= 1/2$

03. find centre and the radius of the circle
$3x^2 + 3y^2 - 18x + 6y + 7 = 0$

$3x^2 + 3y^2 - 18x + 6y + 7 = 0$

$x^2 + y^2 - 6x + 2y + \dfrac{7}{3} = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 \;,\quad 2f = 2 \;,\quad c = 7/3$

$g = -3 \qquad f = 1$

$C \equiv (-g,-f) \equiv (3,-1)$

$R = \sqrt{g^2 + f^2 - c} = \sqrt{9 + 1 - \dfrac{7}{3}} = \sqrt{\dfrac{23}{3}}$

04. $\lim_{x\to 1/4} \dfrac{4x - 1}{2\sqrt{x} - 1}$

$= \lim_{x\to 1/4} \dfrac{4x - 1}{2\sqrt{x} - 1} \cdot \dfrac{2\sqrt{x} + 1}{2\sqrt{x} + 1}$

$= \lim_{x\to 1/4} \dfrac{4x - 1}{4x - 1} \cdot \dfrac{2\sqrt{x} + 1}{1}$

$= \lim_{x\to 1/4} 2\sqrt{x} + 1$

$= 2\sqrt{1/4} + 1$

$= 1 + 1 = 2$

05.

Find the length of latus rectum and equation of directrices of the ellipse $3x^2 + 4y^2 = 1$

$$\frac{x^2}{1/3} + \frac{y^2}{1/4} = 1$$

$a^2 = 1/3 \quad \therefore a = 1/\sqrt{3}$

$b^2 = 1/4 \quad \therefore b = 1/2 \quad a > b$

Eccentricity

$b^2 = a^2(1 - e^2)$

$\frac{1}{4} = \frac{1}{3}(1 - e^2)$

$\frac{3}{4} = 1 - e^2$

$e^2 = 1 - \frac{3}{4}$

$e^2 = \frac{1}{4}$

✓ $e = 1/2$

$\frac{a}{e} = \frac{1/\sqrt{3}}{1/2} = 2/\sqrt{3}$

✓ eq. of directrices : $x = \pm a/e$

$x = \pm 2/\sqrt{3}$

✓ length of latus rectum $= \frac{2b^2}{a} = \frac{2(1/4)}{1/\sqrt{3}}$

$= \frac{1/2}{1/\sqrt{3}}$

$= \sqrt{3}/2$

06.

Find equation of ellipse referred to its principal axes , foci $(\pm 4,0)$ and eccentricity $= 1/3$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

foci $\equiv (\pm ae , 0) \equiv (\pm 4 , 0)$

$\therefore ae = 4$ ...... (1)

$e = \frac{1}{3}$ ..... given

subs in (1) : $a \cdot \frac{1}{3} = 4 \qquad \therefore a = 12$

Now ; $b^2 = a^2(1 - e^2)$

$b^2 = 144\left(1 - \frac{1}{9}\right)$

$b^2 = 144 \times \frac{8}{9} = 128$

Hence ,

equation of the ellipse : $\frac{x^2}{144} + \frac{y^2}{128} = 1$

07. find $\frac{dy}{dx}$ if $y = x^5 . 5^x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x^5 \frac{d}{dx} 5^x + 5^x . \frac{d}{dx} x^5$$

$= x^5 . 5^x . \log 5 + 5^x . (5x^4)$

$= 5^x . (x^5 . \log 5 + 5x^4)$

$= x^4 . 5^x . (x.\log 5 + 5)$

08. $\cot^{-1}(3) + \cot^{-1}\left(\frac{3}{4}\right) = \cot^{-1}\left(\frac{1}{3}\right)$

$= \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{4}{3}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{3} + \frac{4}{3}}{1 - \frac{1}{3} . \frac{4}{3}}\right)$

$= \tan^{-1}\left(\frac{\frac{3 + 12}{9}}{\frac{9 - 4}{9}}\right)$

$= \tan^{-1}\left(\frac{15}{5}\right)$

$= \cot^{-1}\left(\frac{1}{3}\right)$

Q2. (A)

Q - 2A

01. Prove :

$\tan 100 - \tan 65 - \tan 35 = \tan 100 \cdot \tan 65 \cdot \tan 35$

$\tan 100 = \tan(65+35)$

$$\tan 100 = \frac{\tan 65 + \tan 35}{1 - \tan 65 . \tan 35}$$

$\tan 100 - \tan 100.\tan 65.\tan 35 = \tan 65 + \tan 35$

$\tan 100 - \tan 65 - \tan 35 = \tan 100.\tan 65.\tan 35$

.... proved

02. $\tan^{-1}\sqrt{\dfrac{1-x}{1+x}} = \dfrac{1}{2}\cos^{-1}x$

LHS

$$= \tan^{-1}\sqrt{\frac{1-x}{1+x}}$$

Put $x = \cos\theta$

$$= \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \tan^{-1}\sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}}$$

$$= \tan^{-1}\sqrt{\frac{\sin\theta/2}{\cos\theta/2}}$$

$$= \tan^{-1}\tan\theta/2$$

$$= \theta/2$$

$$= \frac{1}{2}\cos^{-1}x$$

03. Prove :

$$\frac{\cos 3A - 2.\cos 5A + \cos 7A}{\cos A - 2.\cos 3A + \cos 5A} = \cos 2A - \sin 2A \cdot \tan 3A$$

LHS

$$= \frac{\cos 7A + \cos 3A - 2.\cos 5A}{\cos 5A + \cos A - 2.\cos 3A}$$

$$= \frac{2\cos\left[\frac{7A+3A}{2}\right].\cos\left[\frac{7A-3A}{2}\right] - 2.\cos 5A}{2\cos\left[\frac{5A+A}{2}\right].\cos\left[\frac{5A-A}{2}\right] - 2.\cos 3A}$$

$$= \frac{2\cos 5A \cdot \cos 2A - 2.\cos 5A}{2\cos 3A \cdot \cos 2A - 2.\cos 3A}$$

$$= \frac{2\cos 5A \cdot \cancel{(\cos 2A - 1)}}{2\cos 3A \cdot \cancel{(\cos 2A - 1)}}$$

$$= \frac{\cos 5A}{\cos 3A}$$

$$= \frac{\cos(3A+2A)}{\cos 3A}$$

$$= \frac{\cos 3A \cdot \cos 2A}{\cos 3A} - \frac{\sin 3A \cdot \sin 2A}{\cos 3A}$$

$$= \cos 2A - \sin 2A \cdot \tan 3A$$

Q - 2B

Q2. (B)

01.

find circle concentric with $x^2 + y^2 - 6x + 60 = 0$ and having circumference 4π

STEP 1

$$x^2 + y^2 - 6x + 60 = 0$$

On comparing with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -6 \quad , \quad 2f = 0$$

$$g = -3 \qquad f = 0$$

$$C \equiv (-g, -f) \equiv (3,0)$$

STEP 2

$$\text{Circumference} = 4\pi$$

$$2\pi r = 4\pi$$

$$r = 2$$

STEP 3

$C(3,0)$, $r = 2$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-3)^2 + (y-0)^2 = 2^2$$

$$x^2 - 6x + 9 + y^2 = 4$$

$$x^2 + y^2 - 6x + 5 = 0$$

02.

find focal distance of point P on the parabola $5y^2 = 12x$ if the abscissa of P is equal to 7

$$5y^2 = 12x$$

$$y^2 = \frac{12x}{5}$$

$$4a = \frac{12}{5}$$

$$a = \frac{3}{5}$$

P(7,y) lies on the parabola

Focal distance of P

$= PS$

$= PM$ .... Focus directrix property

$= x + a$

$= 7 + \frac{3}{5}$

$= \frac{38}{5}$

03. find the equation of the ellipse referred to its principal axis given that $e = \sqrt{3}/2$ and passing through $(6,-4)$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$e = \sqrt{3}/2$$

Now ; $b^2 = a^2(1 - e^2)$

$$b^2 = a^2\left(1 - \frac{3}{4}\right)$$

$$b^2 = \frac{a^2}{4}$$

$$a^2 = 4b^2 \qquad \ldots\ldots \quad (1)$$

Since ellipse is passing through $(6,-4)$, it must satisfy the equation of the ellipse

$$\frac{36}{a^2} + \frac{16}{b^2} = 1 \qquad \ldots\ldots\ldots \quad (2)$$

Solving (1) & (2)

$$\frac{36}{4b^2} + \frac{16}{b^2} = 1$$

$$\frac{9}{b^2} + \frac{16}{b^2} = 1$$

$$b^2 = 25$$

subs in (1) $a^2 = 4(25) = 100$

Hence ,
equation of the ellipse : $\frac{x^2}{100} + \frac{y^2}{25} = 1$

Q - 3A

01. f(x) = $x^2 + 3x + 1$, g(x) = x – 2 .
Find fog^{-1}

STEP 1

$$g(x) = x - 2$$
$$y = x - 2$$
$$x = y + 2$$
$$g^{-1}(x) = x + 2$$

STEP 2

$$f(x) = x^2 + 3x + 1$$

fog–1

$$= f[g^{-1}(x)]$$
$$= g^{-1}(x)\ ^2 + 3\ g^{-1}(x) + 1$$
$$= (x+2)^2 + 3(x + 2) + 1$$
$$= x^2 + 4x + 4 + 3x + 6 + 1$$
$$= x^2 + 7x + 11$$

02.

Solve the following equations using Cramer's Rule

2x – y + 3z = 9 , x + y + z = 6 , x – y + z = 2

$$D = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} \begin{aligned} &= 2(1+1)+1(1-1)+3(-1-1) \\ &= 4 + 0 - 6 \\ &= -2 \end{aligned}$$

$$Dx = \begin{vmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} \begin{aligned} &= 9(1+1)+1(6-2)+3(-6-2) \\ &= 18 + 4 - 24 \\ &= -2 \end{aligned}$$

$$Dy = \begin{vmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix} \begin{aligned} &= 2(6-2)-9(1-1)+3(2-6) \\ &= 8 - 0 - 12 \\ &= -4 \end{aligned}$$

$$D = \begin{vmatrix} 2 & -1 & 9 \\ 1 & 1 & 6 \\ 1 & -1 & 2 \end{vmatrix} \begin{aligned} &= 2(2+6)+1(2-6)+9(-1-1) \\ &= 16 - 4 - 18 \\ &= -6 \end{aligned}$$

$$x = \frac{Dx}{D} = 1 \ ; \quad y = \frac{Dy}{D} = 2 \ ; \quad z = \frac{Dz}{D} = 3$$

SS {1 , 2 , 3}

03.

Find equation of hyperbola whose foci are (0,±12) and the length of latus rectum is 36

Let the equation of the hyperbola be

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

$$\text{foci} \equiv (0, \pm be) = (0, \pm 12)$$

$$be = 12 \ ...... \ (1)$$

length of latus rectum

$$\frac{2a^2}{b} = 36$$

$$a^2 = 18b \ ....(2)$$

$$\text{Now } a^2 = b^2(e^2 - 1)$$

$$18b = b^2\left(\left(\frac{12}{b}\right)^2 - 1\right)$$

$$18b = b^2\left(\frac{144 - b^2}{b^2}\right)$$

$$18b = 144 - b^2$$

$$b^2 + 18b - 144 = 0$$

$$(b + 24)(b - 6) = 0$$

$$b \neq 24 \ , \ b = 6$$

subs in (2)

$$a^2 = 18(6) = 108$$

∴ the equation of the hyperbola be

$$\frac{y^2}{36} - \frac{x^2}{108} = 1$$

Q3 (B)

Q - 3B

01.

$$\lim_{x \to 0} \frac{\log(4+x) - \log(4-x)}{\sin x}$$

Dividing Numerator & Denominator by x ,
as $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\dfrac{\log(4+x) - \log(4-x)}{x}}{\dfrac{\sin x}{x}}$$

$$= \lim_{x \to 0} \frac{\log(4+x) - \log(4-x)}{x}$$

$$= \lim_{x \to 0} \frac{1}{x} \log\left[\frac{4+x}{4-x}\right]$$

$$= \lim_{x \to 0} \frac{\log\left[\dfrac{4+x}{4-x}\right]}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(\dfrac{\frac{4+x}{4}}{\frac{4-x}{4}}\right)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(\dfrac{1+\frac{x}{4}}{1-\frac{x}{4}}\right)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left[1+\frac{x}{4}\right] - \log\left[1-\frac{x}{4}\right]}{x}$$

$$= \lim_{x \to 0} \frac{\log\left[1+\frac{x}{4}\right]}{x} - \frac{\log\left[1-\frac{x}{4}\right]}{x}$$

$$= \lim_{x \to 0} \frac{1}{4}\frac{\log\left[1+\frac{x}{4}\right]}{\frac{x}{4}} + \frac{1}{4}\frac{\log\left[1-\frac{x}{4}\right]}{-\frac{x}{4}}$$

$$= \frac{1}{4}(1) + \frac{1}{4}(1) = \frac{1}{2}$$

02. the total cost of x pencils is given by $c = 15 + 28x - x^2$. Find x when marginal cost is 20 . Find the average cost at this value of x

SOLUTION :

Marginal cost = 20

$$\frac{dC}{dx} = 20$$

$$28 - 2x = 20$$

$$8 = 2x$$

$$x = 4$$

Average cost at x = 4

$$= \frac{C}{x}$$

$$= \frac{15 + 28x - x^2}{x}$$

$$= \frac{15}{x} + 28 - x$$

Put x = 4

$$= \frac{15}{4} + 28 - 4$$

$$= \frac{15}{4} + 24$$

$$= \frac{15 + 96}{4}$$

$$= \frac{111}{4}$$

03. $y = \sin^3 3x.\, e^{\sqrt{x}} + \log \dfrac{x+1}{\sqrt{x^2+1}}$

STEP 1

$$\frac{d}{dx} \sin^3 3x.\, e^{\sqrt{x}}$$

$$= \sin^3 3x. \frac{d}{dx} e^{\sqrt{x}} + e^{\sqrt{x}} \frac{d}{dx} \sin^3 3x$$

$$= \sin^3 3x.\, e^{\sqrt{x}} \frac{d}{dx} \sqrt{x} + e^{\sqrt{x}}\, 3 \sin^2 3x \frac{d}{dx} \sin 3x$$

$$= \sin^3 3x.\, e^{\sqrt{x}} \frac{1}{2\sqrt{x}} + e^{\sqrt{x}}\, 3\sin^2 3x\, .\cos 3x \frac{d}{dx} 3x$$

$$= \sin^3 3x.\, e^{\sqrt{x}} \frac{1}{2\sqrt{x}} + e^{\sqrt{x}}\, 3 \sin^2 3x\, .\, \cos 3x\;\; 3$$

$$= \frac{e^{\sqrt{x}}.\sin^3 3x.}{2\sqrt{x}} + 9\, e^{\sqrt{x}} \sin^2 3x\, .\, \cos 3x$$

$$= e^{\sqrt{x}}.\sin^2 3x \left(\frac{\sin 3x}{2\sqrt{x}} + 9.\, \cos 3x \right)$$

STEP 2 :

$$\frac{d}{dx} \log \frac{x+1}{\sqrt{x^2+1}}$$

$$= \frac{d}{dx} \left[\log (x+1) - \log \sqrt{x^2+1} \right]$$

$$= \frac{d}{dx} \left[\log (x+1) \frac{-1}{2} \log (x^2+1) \right]$$

$$= \frac{1}{x+1} \frac{d}{dx} (x+1) - \frac{1}{2} \frac{1}{x^2+1} \frac{d}{dx} (x^2+1)$$

$$= \frac{1}{x+1} - \frac{1}{2} \frac{1}{x^2+1} 2x$$

$$= \frac{1}{x+1} - \frac{x}{x^2+1}$$

STEP 3 :

$$\frac{dy}{dx} = e^{\sqrt{x}}.\sin^2 3x \left(\frac{\sin 3x}{2\sqrt{x}} + 9\cos 3x \right) + \frac{1}{x+1} - \frac{x}{x^2+1}$$

SECTION - II

Q - 4

Q4

01. in a sociological study of 500 persons , 300 wives married . 250 were successful executives , 198 successful executives were married . Is the data consistent

A ≡ married B ≡ successful

	B	β	TOTAL
A	**(AB) = 198**	**(Aβ) = 102**	**(A) = 300**
α	**(αB) = 52**	**(αβ) = 148**	**(α) = 200**
	(B) = 250	**(β) = 250**	**N = 500**

the data is CONSISTENT

02.

if $\Sigma p_0q_0 = 120$, $\Sigma p_0q_1 = 200$, $\Sigma p_1q_1 = 300$ and $P_{01}(L) = 150$. Find $P_{01}(M\text{-}E)$

$$P_{01}(L) = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100$$

$$150 = \frac{\Sigma p_1q_0}{120} \times 100$$

$$\Sigma p_1q_0 = \frac{150 \times 120}{100} = 180$$

$$P_{01}(ME) = \frac{\Sigma p_1q_0 + \Sigma p_1q_1}{\Sigma p_0q_0 + \Sigma p_0q_1} \times 100$$

$$= \frac{180 + 300}{120 + 200} \times 100$$

$$= \frac{480}{320} \times 100 = 150$$

03.

let A and B be two events such that P(A) = 0.3 . $P(A\cup B) = 0.8$. If A and B are independent events , then find P(B)

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A\cup B) = P(A) + P(B) - P(A).P(B)$

.... A & B are independent

$0.8 = 0.3 + P(B) - 0.3P(B)$

$0.5 = 0.7P(B)$

$P(B) = 5/7$

04. $^{12}C_5 + 2.^{12}C_4 + {^{12}C_3} = {^{14}C_x}$

$\left({^{12}C_5} + {^{12}C_4}\right) + \left({^{12}C_4} + {^{12}C_3}\right) = {^{14}C_x}$

${^{13}C_5} + {^{13}C_4} = {^{14}C_x}$

${^{14}C_5} = {^{14}C_x}$

${^{14}C_5} = {^{14}C_9} = {^{14}C_x}$

x = 5 or 9

05.

the index number for the year 2004 taking 2002 as base year was found to be 120 . Find the missing details if $\Sigma p_0 = 320$

Commodity	A	B	C	D	E	F
p_o(2002)	40	60	20	x	50	110
p_1(2004)	50	70	30	85	y	115

$\Sigma p_0 = 320$,

$280 + x = 320$ ∴ x = 40

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

$$120 = \frac{350 + y}{320} \times 100$$

$$350 + y = \frac{120 \times 320}{100}$$

$350 + y = 384$ ∴ y = 34

06. for the following data , find the value of x if the Laspeyre's price index number is equal to Paasche's price index number

Commodity	p_0	q_0	p_1	q_1
A	3	x	2	5
B	4	6	3	5

p_1q_0	p_1q_1	p_0q_0	p_0q_1
2x	10	3x	15
18	15	24	20
2x+18	25	3x+24	35

$P_{01}(L) = P_{01}(P)$

$$\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \cancel{100} = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times \cancel{100}$$

$$\frac{2x + 18}{3x + 24} = \frac{25}{35}$$

$7(2x + 18) = 5(3x + 24)$ ∴ x = 6

07. two unbiased dice are rolled . Find the probability that the sum of numbers on the upper most faces is divisible by 2 or 4

NOTE : we have been solving such sums using ADDITION THOREM as done in mock paper 3 in a similar sum . However here I have solved using CLASSICAL DEFINITION

Exp : two unbiased dice are rolled

n(S) = 36

E ≡ sum of numbers on the upper most faces is divisible by 2 or 4

≡ sum of numbers is 2 , 4, 6 , 8 , 10 or 12

≡ (1,1) , (1,3) , (2,2) , (3,1) , (1,5) , (2,4) , (3,3) , (4,2) , (5,1) , (2,6) , (3,5) , (4,4) , (5,3) , (6,2) , (4,6) , (5,5) , (6,4) , (6,6)

n(E) = 18

$$P(E) = \frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

08.

Compute 3 yearly moving average values

Year :	2004	2005	2006	2007	2008	2009
IMR :	114	97	80	74	68	58

Year : 2010

IMR : 49

Year	IMR	3 year MOVING TOTAL T	3 year MOVING AVG $^{T}/_{3}$
2004	114		
2005	97	114 + 97 + 80 = 291	291/3 = 97
2006	80	97 + 80 + 74 = 251	251/3 = 83.67
2007	74	80 + 74 + 68 = 222	222/3 = 74
2008	68	74 + 68 + 58 = 200	200/3 = 66.67
2009	58	68 + 58 + 49 = 175	175/3 = 58.33
2010	49		

Q - 5A

Q5(A)

01.

(AB) = 128 ;(αB) = 384 ;(Aβ)=24 ;(αβ) = 72 .

Examine whether attributes A and B are independent or not

1) (AB)(αβ) = 128 x 72 = 9216

2) (Aβ)(αB) = 24 x 384 = 9216

3) (AB)(αβ) = (Aβ)(αB)

4) attributes A and B are independent

02.

Two students appear for an examination , their chances of passing the examination being 0.7 and 0.8 respectively . Find the probability that only one of them passes the examination

A ≡ student A passes the exam , P(A)=0.7

B ≡ student A passes the exam , P(B)=0.8

E ≡ only one of them passes the examination

E ≡ $(A\cup B) - (A\cap B)$

$P(E) = P(A\cup B) - P(A\cap B)$

$= P(A) + P(B) - P(A\cap B) - P(A\cap B)$

$= P(A) + P(B) - 2P(A\cap B)$

$= P(A) + P(B) - 2P(A)P(B)$

..... A & B are independent

$= 0.7 + 0.8 - 2(0.7)(0.8)$

$= 1.5 - 1.12$

$= 0.38$

03. Obtain the trend component of the following time series of production using 4 – yearly moving average

Year	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
Production	12	15	18	17	16	20	23	22	24	25

Year	IMR	4 Year Moving total	4 year moving total Centered (T)	4 year moving avg Centered (T/8)
1971	12			
1972	15			
		12+15+18+17 = 62		
1973	18		62 + 66 = 128	$^{128}/_{8}$ = 16
		15+18+17+16 = 66		
1974	17		66 + 71 = 137	$^{137}/_{8}$ = 17.125
		18+17+16+20 = 71		
1975	16		71 + 76 = 147	$^{147}/_{8}$ = 18.375
		17+16+20+23 = 76		
1976	20		76 + 81 = 157	$^{157}/_{8}$ = 19.625
		16+20+23+22 = 81		
1977	23		81 + 89 = 170	$^{170}/_{8}$ = 21.25
		20+23+22+24 = 89		
1978	22		89 + 94 = 183	$^{183}/_{8}$ = 22.875
		23+22+24+25 = 94		
1979	24			
1980	25			

Q5.(B)

Q - 5B

01. Calculate Walsch's Price Index Number

p_o	q_o	p_1	q_1	$\sqrt{q_0 q_1}$	$p_1.\sqrt{q_o q_1}$	$p_o\sqrt{q_0 q_1}$
10	12	40	3	6	240	60
20	2	25	8	4	100	80
30	3	50	27	9	450	270
60	9	90	36	18	1620	1080
					2410	1490

$$P_{01(W)} = \frac{\Sigma p_1.\sqrt{q_o q_1}}{\Sigma p_o\sqrt{q_0 q_1}} \times 100 = \frac{2410 \times 100}{1490}$$

$$= 161.7$$

LOG CALC

3.3820
–3.1732
AL(0.2088)
1.617

02. How many 5 digit numbers can be formed using digits 0,1,2,3,4,5 which are divisible by 3 without repeating the digits

(NOTE : divisibility test for 3 is sum of digits have to be multiple of 3)

Case1 : 5 digit numbers formed using digits 0 , 1 , 2 , 4 , 5

Ten thousand place can be filled by any of the 4 digits (excluding 0) in ${}^4P_1$ ways

Remaining 4 places can be filled by the remaining 4 digits in ${}^4P_4 = 4!$ ways

By fundamental principle of multiplication , nos. formed $= {}^4P_1 \times 4! = 96$

Case2 : 5 digit numbers formed using digits 1 , 2 , 3 , 4 , 5

5 places can be filled by the 5 digits in ${}^5P_5 = 5!$ways

By fundamental principle of multiplication , nos. formed $= 5! = 120$

Therefore ; By fundamental principle of ADDITION ,

Total numbers $= 96 + 120 = 216$

03.

	Fever	No Fever	Total
Quinine	20	792	812
No Quinine	220	2216	2436

Discover the usefulness of quinine in checking malaria

NOTE : We will find yules coefficient of association between Quinine and no Fever to examine its effectiveness in preventing Fever and hence

A : person is administered with 'QUININE'
B : person is suffereing from 'NO FEVER

		No Fever B	Fever β
Quinine	A	$(AB) = 792$	$(A\beta) = 20$
No Quiniine	α	$(\alpha B) = 2216$	$(\alpha\beta) = 220$

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$= \frac{792 \times 220 - 20 \times 2216}{792 \times 220 + 20 \times 2216}$$

$$= \frac{174240 - 44320}{174240 + 44320}$$

$$= \frac{129920}{218560}$$

$$= \frac{12992}{21856}$$

$$= 0.5943$$

LOG CALC.
$4.\ 1137$
$-\ 4.\ 3397$
AL $\bar{1}.\ 7740$
$0.\ 5943$

COMMENT

There is significant amount of positive asscociation between A and B and hence Quinine is effective in preventing Fever

Q - 6A

Q6. (A)

01.

Compute the standard deviation for the following data

CI :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
f :	11	15	25	12	7

CI	f	x	$u = \frac{x - 25}{10}$	fu	fu^2
0 – 10	11	5	–2	–22	44
10 – 20	15	15	–1	–15	15
20 – 30	25	25	0	0	0
30 – 40	12	35	1	12	12
40 – 50	7	45	2	14	28
	70			–11	99

$$\sigma u = \sqrt{\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{99}{70} - \left(\frac{-11}{70}\right)^2}$$

$$= \sqrt{\frac{6930 - 121}{70^2}}$$

$$= \sqrt{\frac{6809}{4900}}$$

$$= 1.179$$

$$\sigma x = 10 \times \sigma u = 11.79$$

LOG CALC

3.8331
– 3.6902
0.1429
2
AL(0.0715)
1.179

02. In how many ways can a committee of 3 ladies and 4 gents be chosen from 8 ladies and 7 gents . What is the number of ways if Miss X refuses if Mr Y is a member .

Case 1 : Mr Y is a member

Since Mr Y is a member , the remaining 3 gents have to be selected from the remaining 6 gents . This can be done in ${}^6C_3$ ways

Since Mr Y is a member , Miss X will not the member . Therefore the 3 ladies have to be selected from the remaining 7 ladies . This can be done in ${}^7C_3$ ways

By fundamental principle of Multiplication

No of ways of forming such a committee $= {}^6C_3 \times {}^7C_3 = 20 \times 35 = 700$

Case 2 : Mr Y is NOT a member

Since Mr Y is not a member , the 4 gents have to be selected from the remaining 6 gents . This can be done in ${}^6C_4$ ways

Having done that ,

the 3 ladies have to be selected from the remaining 8 ladies . This can be done in ${}^8C_3$ ways

By fundamental principle of Multiplication

No of ways of forming such a committee $= {}^6C_4 \times {}^8C_3 = {}^6C_2 \times {}^8C_3$

$= 15 \times 56 = 840$

By fundamental principle of addition

Total ways of forming the committee $= 700 + 840 = 1540$

03. If there are 12 points in a plane out of which 'p' points are collinear , find the value of 'p' for which 185 triangles can be obtained by joining these 12 points

12 points

3 points define a triangle

$\therefore$ number of Δ's that can be drawn $= {}^{12}C_3$

$= 220$

But p points are collinear

Number of triangles wrongly counted in these

p collinear points $= {}^{p}C_3$ instead of 0

Hence

actual triangles that can be drawn

$220 - {}^{p}C_3 = 185$ ... given

${}^{p}C_3 = 35$

$\dfrac{p(p-1)(p-2)}{3.2.1} = 35$

$p(p-1)(p-2) = 210$

$p(p-1)(p-2) = 7.6.5$

$\therefore$ On Comparison $n = 7$

Q - 6B

Q6. (B)

01. there are two urns A and B . A contains 3 white & 5 red balls . B contains 2 white & 4 red balls . One urn is selected at random & a ball is drawn from it at random . Find the probability that the ball drawn is white

exp : One urn is selected at random & a ball is drawn from it at random

$E \equiv$ ball drawn is white

$\equiv E_1 \cup E_2$

$E_1 \equiv$ urn A is selected AND a white ball is drawn

$E_1 \equiv A \cap B$

$P(E1) = P(A \cap B)$

$= P(A) \times P(B \mid A)$

$= \dfrac{1}{2} \times \dfrac{3}{8} = \dfrac{3}{16}$

$E_2 \equiv$ urn B is selected AND a white ball is drawn

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$= P(A) \times P(B \mid A)$

$= \dfrac{1}{2} \times \dfrac{2}{6} = \dfrac{1}{6}$

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ .... Mutually Exclusive

$= \dfrac{3}{16} + \dfrac{1}{6}$

$= \dfrac{9 + 8}{48}$

$= \dfrac{17}{48}$

02.

fit a trend line using method of least squares . Also obtain trend value for the year 2006

Year :	2000	2001	2002	2003	2004	2005
Sales:	105	118	125	130	150	172

t	y	u = (t–2002.5)x2	u^2	yu
2000	105	–5	25	–525
2001	118	–3	9	–354
2002	125	–1	1	–125
2003	130	1	1	130
2004	150	3	9	450
2005	172	5	25	860
	800	0	70	–1004+1440 436

trend line :

$y = a + bu \longrightarrow yu = au + bu^2$

$\Sigma y = na + b\Sigma u$ $\qquad \Sigma yu = a\Sigma u + b\Sigma u^2$

$800 = 6a$ $\qquad 436 = b(70)$

$a = 133.33$ $\qquad b = 6.23$

$y = 133.33 + 6.23u$; $\quad u = (t - 2002.5)x2$

trend value for the year 2006

$u = (2006 - 2002.5)x2 = 7$

$y = 133.33 + 6.23(7) = 176.94$

03. Find the cost of Living Index number

Group	p_0	p_1	w	$I = \frac{p_1}{P_0} \times 100$	Iw
Food	12	60	25	$\frac{60}{12} \times 100 = 500$	12500
Clothing	10	45	20	$\frac{45}{10} \times 100 = 450$	9000
Fuel & Light	20	35	15	$\frac{35}{20} \times 100 = 175$	2625
House Rent	25	20	30	$\frac{20}{25} \times 100 = 80$	2400
Misc	16	48	10	$\frac{48}{16} \times 100 = 300$	3000
			$\Sigma w = 100$		$\Sigma Iw = 29525$

$$CLI = \frac{\Sigma Iw}{\Sigma w} = \frac{29525}{100} = 295.25$$

In 2007 ; CLI = 100 ; Expenditure = 10,000/-

In 2008 ; CLI = 295.25 ; Expenditure = $\frac{295.25 \times 10000}{100}$ = 29,525/-